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SPHERICALLY SYMMETRIC ESCAPE OF A SELF-GRAVITATING IDEAL GAS INTO A VACUUM†

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The spherically symmetric flows of an ideal gas are considered assuming that Newtonian gravitation acts on the mass of gas. The problem of the decay of a special discontinuity is investigated by the method of characteristic series, and exact solutions of the initial boundary-value problems of the non-linear integrodifferential partial-differential system are constructed in the form of converging series. It is proved that the particles of gas on the free gas–vacuum surface move as particles in the field of attraction of a material point situated at the centre of symmetry and having a mass equal to the initial mass of gas. In the case when the gas and the vacuum are continuously adjacent to one another one can prove a theorem of the existence and uniqueness of the solution for only rational adiabatic indices, and one can show that for certain values of the gas-dynamic parameters the gas sphere disperses to infinity, while in other cases the gas–vacuum boundary stops and the mass of gas begins to collapse. Singularities of the solution on this boundary only appear at the instant of focusing, which can be treated as the instant when the whole mass of gas collapses into the centre of gravity.

Problems similar to one mentioned have been considered previously, but without taking gravitation into account. Using characteristic series in the neighbourhood of the boundary Γ_1 two-dimensional flows [1] and three-dimensional flows of an ideal gas adjacent to a region of a gas at rest were constructed. The decay of an arbitrary discontinuity on a curvilinear surface when the discontinuity in the gas density was greater than zero on both sides of the surface was considered in [2]. By analysing the first terms of certain asymptotic expansions it was concluded [3] that the free surface Γ_0 moves with constant velocity for a certain time. The flow that occurs as a result of the collapse of a one-dimensional cavity was investigated in [4]. When $1<\gamma<3$ the solution was constructed in the form of converging characteristic series in the region from Γ_1 to Γ_0 inclusive, and it was proved that the surface Γ_0 moves with constant velocity for a certain time. This result was generalized to the case of two- and three-dimensional flows in [5], and to three-dimensional flows when external mass forces act in [6].

In the case of a gravitating gas, adiabatic motions with uniform deformation, when the velocities are linear functions of the coordinates, have been investigated in a large number of publications. An accurate solution for the spherically symmetric motion of a gravitating gas with varying density was obtained in explicit form in [7]. The dynamics of the adiabatic motions of a gravitating gaseous ellipsoid were investigated in [8].

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1. FORMULATION OF THE PROBLEM OF THE DECAY OF A SPECIAL DISCONTINUITY

Suppose that at an instant $t=0$ a sphere Γ of radius r^0 isolates from a vacuum an ideal polytropic gas which gravitates in accordance with Newton's law. At the instant $t=0$ we know the distributions of the gas parameters in the sphere: $u=u_0(t)$ is the gas velocity, $\rho=\rho_0(x)$ is the gas density, and $S=S_0(x)$ is the entropy, where x is the distance to the centre of the sphere Γ, $0\leqslant x<r^0$. The functions u_0, S_0, and ρ_0 are assumed to be analytic, and the gas density is assumed to be greater than zero everywhere in the sphere, including $\rho_0(x)|_\Gamma>0$.

At the instant $t=0$ motion of the gas begins, determined by these distributions u_0, S_0, and ρ_0 and which will henceforth be called background flow. Simultaneously, at the instant $t=0$, the surface Γ is instantaneously demolished and part of the gravitating ideal gas begins to disperse into the vacuum. The perturbations that occur in the background flow as a result of the instantaneous removal of the surface Γ, propagate in the gas in the form of a rarefaction wave, separated from the background flow by the boundary Γ_1, which is a surface of weak discontinuity. The rarefaction wave touches the vacuum from the other side: $\rho|_{\Gamma_0}=0$, where Γ_0 is the free surface which separates the rarefaction wave from the vacuum. It is required to construct both the background flow and the rarefaction wave, and also to obtain the laws of motion of Γ and Γ_0.

The spherically symmetric flows of the gas considered are described by the following system of equations [9]

$$\rho_t+(\rho u_x+2\rho u/x=0$$
$$u_t+uu_x+\frac{1}{\rho}p_x=F(x,\ t),\ F(t,\ x)=-4\pi\frac{G}{x^2}\int_0^x r^2\rho(r,\ t)dr \tag{1.1}$$
$$S_t+uS_x=0,\ p=S^2\rho^\gamma/\gamma,\ \gamma=\text{const}>1$$

where p is the pressure and G is the gravitational constant.

To simplify our further analysis we will change from the system of integro-differential equations (1.1) to a system of differential equations by introducing an additional unknown function $F(x,t)$. Differentiating F with respect to x and t and taking the equation of continuity into account, we obtain two differential equations for F

$$F_x=-2x^{-1}F+\ 4\pi G\rho, F_t=4\pi G\rho u \tag{1.2}$$

We can also take $M=-x^2G^{-1}F(t,\ x)$ as the new unknown function, and the equations for M will then have the form

$$M_x=4\pi x^2\rho,\ M_t=-4\pi x^2\rho u \tag{1.3}$$

We will use the function F and Eqs (1.2) in the problem of the decay of the discontinuity. Moreover, we will also use Eqs (1.3). System (1.1) and (1.2) obtained is overdefined: there are five equations for four unknowns, but it can be shown by cross differentiation that it is consistent.

It is convenient to take $\sigma=\rho^{(\gamma-1)/2}$ as the unknown function instead of ρ. To construct the background flow we need to solve the Cauchy problem for the system considered with the following initial data

$$t=0,\ u=u_0(x_x),\ S=S_0(x),\ \sigma=\sigma_0(x) \tag{1.4}$$
$$F=F_0(x)=-4\pi\frac{G}{x^2}\int_0^x r^2\rho_0(r)dr$$

If $\rho_0(x)$ is an analytic function, it can be shown that $F_0(x)$ is also an analytic function, which has no discontinuity at $x=0$. Since the system considered is a system of the Cauchy–Kovalevskii type while the initial data are analytic functions, the Cauchy problem has an analytic solution for small t [10], which can be represented, for example, in the form of converging series in powers of t with coefficients which are analytic functions of x. Using this solution one can uniquely construct (for example, in the form of series in powers of t) $x_1(t)$ and

$$\sigma\big|_{\Gamma_1}=\sigma^0(t),\ u\big|_{\Gamma_1}=u^0(t),\ S\big|_{\Gamma_1}=S^0(t) \tag{1.5}$$

Here $x_1(t)$ is the law of motion of the surface of the weak discontinuity Γ_1, which is the sonic characteristic of the background flow, and σ^0, u^0, S^0 are the values of the gas-dynamic parameters in it. Henceforth we will assume that we know the following: the background flow, the surface Γ_1, and σ^0, u^0, S^0. To construct the rarefaction wave we will make the following replacement of variables: we will take t and σ as the independent variables, and we will take x, u, S and F as the unknown functions. The Jacobian of this transformation $J=x_\sigma$. We then obtain the following system of equations

$$\begin{aligned}
&x_t=u+\frac{\gamma-1}{2}u_0\sigma+(\gamma-1)x_\sigma\frac{u\sigma}{x}\\
&x_\sigma u_t-\frac{\gamma-1}{2}u_\sigma^2\sigma-(\gamma-1)x_\sigma u_\sigma\frac{u\sigma}{x}+\frac{2}{\gamma-1}S^2\sigma+\frac{2}{\gamma}SS_\sigma\sigma^2=x_\sigma F\\
&x_\sigma S_t+(u-x_t)S_\sigma=0\\
&F_t=-2x^{-1}x_tF+4\pi G(u-x_t)\sigma^{2/(\gamma-1)}
\end{aligned} \tag{1.6}$$

The flow in the region between Γ_1 and Γ_0 (the rarefaction wave) will be constructed as the solution of system of (1.6) with the data on the characteristic Γ_1 (1.5). Since Γ_1 is a characteristic of multiplicity one, to obtain a unique locally analytic solution we need to specify one additional condition [11]. If the surface Γ is removed slowly, the following relation serves as this condition in the space of variables (σ, t) [4–6]

$$x(0,\ \sigma)=r^0 \tag{1.7}$$

2. CONSTRUCTION OF THE RAREFACTION WAVE

Theorem 1. When $0<t<t_0$, in a certain neighbourhood of Γ_1 there is a unique locally analytic solution of problem (1.5)–(1.7) on the decay of the discontinuity.

The proof of the theorem reduces [4–6] to the corresponding analogue of the Cauchy–Kovalevskii theorem [11].

To investigate the question of whether the surface Γ_0 lies in the region in which this solution is applicable, we will expand the solution of problem (1.5)–(1.7) in series in powers of t

$$\mathbf{f}(t,\ \sigma)=\sum\mathbf{f}_k(\sigma)t^k/k!\ \ \mathbf{f}=\{x,\ u,\ S,\ F\} \tag{2.1}$$

which, for small t, is possible in view of the analytic form of the solution of the problem of the decay in a certain neighbourhood of Γ_1. Here and henceforth the summation is carried out over k from zero to infinity.

We will put $t=0$ in (1.6) and, taking (1.7) into account, we will have

$$x_1=-2\alpha\sigma S_0+u_*,\ u_0=-\frac{2}{\gamma-1}\sigma S_0+u_*,\ S_0=S_{00}=S_0(r^0)$$

$$u_* = \frac{2}{\gamma-1} S_0(r^0)\sigma_0(r^0) + u_0(r^0),\ 2\alpha = \frac{\gamma+1}{\gamma-1}$$

$$F_1 = \frac{2}{r^0} F_0(u_* - 2\alpha S_0 \sigma) - 4\pi G S_0 \sigma^{2\alpha}$$

$$F_0 = -4\pi \frac{G}{(r^0)^2} \int_0^{r_0} r^2 \rho_0(r) dr$$

We differentiate (1.6) k times with respect to t, put $t=0$, and taking (1.7) and the expressions previously obtained for $f_1(\sigma)$ $(0 \leqslant 1 < k)$ into account, we have

$$x_{k+1} = u_k + \frac{\gamma-1}{2} \sigma u_{k\sigma} + G_{1k}(\sigma)$$

$$\sigma u_{k\sigma} - \alpha k u_k = G_{2k}(\sigma),\ \sigma S_{k\sigma} - 2\alpha k S_k = G_{3k}(\sigma),\ F_{k+1} = G_{4k}(\sigma)$$

Here $G_{1k}, \ldots, G_{4k}$ are functions which depend on $f_1(\sigma)$ $(0 \leqslant 1 < k)$, but they will not be given here in view of their complexity.

Integrating the second and third equations of the system we obtain

$$u_k = \sigma^{\alpha k}(u_{k0} + \int G_{2k}(\sigma)\sigma^{-\alpha k-1} d\sigma)$$

$$S_k = \sigma^{2\alpha k}(S_{k0} + \int G_{3k}(\sigma)\sigma^{-2\alpha k-1} d\sigma) \tag{2.2}$$

The arbitrary constants u_{k0} and S_{k0} are found from conditions (1.5). To do this we substitute $\sigma^0(t)$ into the right-hand side of (2.2), and $u^0(t)$ and $S^0(t)$ into the left-hand sides. Expanding the expressions obtained in powers of t and equating the coefficients of like powers we obtain relations from which u_{k0} and S_{k0} are uniquely determined.

Lemma. When $1 < \gamma < 3$ the coefficients of series (2.1) when $k \geqslant 1$ have the form

$$S_k = \sigma P_{1k}(\sigma,\ \sigma^\lambda,\ \sigma \ln \sigma), \qquad F_k = a_k + \sigma P_{2k}(\sigma,\ \sigma^\lambda,\ \sigma \ln \sigma)$$

$$u_k = a_{k-1} + \sigma P_{3k}(\sigma,\ \sigma^\lambda,\ \sigma \ln \sigma),\ x_{k+1} = a_{k-1} + \sigma P_{4k}(\sigma,\ \sigma^\lambda,\ \sigma \ln \sigma)$$

where $P_{1k}, \ldots, P_{4k}$ are polynomials of the arguments indicated, and $\lambda > 0$, $a_k = \text{const}$.

The proof of the lemma is similar to the corresponding proof from [4–6] and is carried out by induction over k. It is first proved that $G_{ik}(\sigma)$ possess the required structure, and it is then shown by direct integration that u_k possess the structure indicated.

On the basis of the lemma we can assert that the structure of the solution which specifies the rarefaction wave, is as follows:

$$S = \sigma S^1(t,\ \sigma), \qquad x = x^0(t) + \sigma x^1(t,\ \sigma)$$

$$u = u^0(t) + \sigma u^1(t,\ \sigma), \qquad F = F^0(t) + \sigma F^1(t,\ \sigma)$$

Here

$$F^0(t) = \sum a_k t^k / k!, \quad u^0(t) = \sum a_{k+1} t^{k+1} / (k+1)!$$

$$x^0(t) = \sum a_{k+2} t^{k+2} / (k+2)!$$

The convergence of the series for $F^0(t)$, $u^0(t)$, $x^0(t)$, like the convergence of all the series (2.1), is established by the following theorem.

Theorem 2. For $1<\gamma<3$ when $0<t<t_0$ the region of convergence of series (2.1), and also of the series which specify f_t and f_σ, covers the whole flow region from Γ_1 to Γ_0, inclusive. The law of motion of $\Gamma_0: x=x^0(t)$ is then found from the solution of the auxiliary problem $x_t^0=u^0(t)$, $x^0(0)=r^0$

$$u_t^0 = F^0(t),\ u^0(0)=u_* = \frac{2}{\gamma-1}S_0(r^0)\sigma_0(r^0)+u_0(r^0) \tag{2.3}$$

$$F_t^0 = -\frac{2u^0F^0}{x^0(t)},\ F^0(0)=F_0=-4\pi\frac{G}{(r^0)^2}\int_0^{r^0} r^2\rho_0(r)dr$$

and the initial value of the entropy $S|_{\Gamma_0}=S_0(x)|_{\Gamma}=S_0(r^0)$ is conserved on the surface Γ_0.

The proof of the theorem is similar to the proof in [4–6].

An analysis of the coefficients of series (2.1) shows that $x^0(t)$ can also be obtained without constructing the whole solution of problem (1.5)–(1.7). It is sufficient to construct the solution of the auxiliary problem (2.3) in the form of a formal series in powers of t. Since $x^0(0)=r^0>0$, problem (2.3) has a unique locally analytic solution, which once again proves the convergence of the series specifying $x^0(t)$. A detailed investigation of problem (2.3) will be carried out below.

3. THE PROBLEM OF A GAS CONTINUOUSLY ADJACENT TO A VACUUM

In order to determine the instant of time up to which the law of motion of Γ_0 is conserved, we will investigate the problem of a gas that is continuously adjacent to a vacuum. If we have the solution of the problem of the decay of a discontinuity, i.e. if we know, in particular, the quantities $\sigma(t_0, x)$, $u(t_0, x)$, $S(t_0, x)$, and $\sigma(t_0, x)|_{\Gamma_0}=0$ at the instant $t=t_0>0$, we can postulate the Cauchy problem at $t=t_0$ with these initial data for system (1.1) and (1.3). If the solution of this problem exists, we can use it to determine the law of motion of Γ_0 in implicit form $\sigma(t, x)=0$ when $t>t_0$. Here it is natural to assume that the perturbations that occur from the focusing of the weak discontinuity or from possible strong discontinuities in the middle part of the flow, do not reach Γ_0.

Suppose $x=x_0(t)$ is the law of motion of the free surface Γ_0, obtained from the solution of system (2.3). We will introduce the new independent variable $z=x-x_0(t)$, i.e. we will take the surface Γ_0 as the coordinate axis $z=0$. System (1.1) and (1.3) can then be rewritten in the form

$$\sigma_t+(u-x_{0t})\sigma_z+\frac{\gamma-1}{2}u_z\sigma+(\gamma-1)\frac{u\sigma}{z+x_0}=0 \tag{3.1}$$

$$u_t+(u-x_{0t})u_z+\frac{2}{\gamma-1}S^2\sigma\sigma_z+\frac{2}{\gamma}SS_z\sigma^2+\frac{GM}{(z+x_0)^2}=0$$

$$S_t+(u-x_{0t})S_z=0,\ M_z=4\pi(z+x_0)^2\sigma^{2/(\gamma-1)}$$

We specified the following conditions for system (3.1) on the surface Γ_0 for $z=0$

$$\sigma(t,\ 0)=0,\ u(t,\ 0)=u^0(t),\ S(t,\ 0)=S_{00},\ M(t,\ 0)=M_{00} \tag{3.2}$$

Here M_{00} is the initial mass of gas $S_{00}=S(r^0)$, and $u^0(t)$ is the velocity of motion of the surface Γ_0 in the problem of the decay of the discontinuity. Problem (3.1), (3.2) is a characteristic Cauchy problem, and the multiplicity of the characteristic $z=0$ is three. Hence, for the solution to be unique it is necessary [11] to specify the initial data

$$\sigma(t_0,\ z)=\sigma^0(z),\ u(t_0,\ z)=u^0(z),\ S(t_0,\ z)=S^0(z) \tag{3.3}$$

which agrees at the point $t=t_0,\ z=0$ with the data of (3.2).

System (3.1) is not analytic for arbitrary $\gamma>1$, so that we cannot construct a solution in the neighbourhood of Γ_0 which uses analogues of the Cauchy–Kovalevskii theorem. Nevertheless, we can write and investigate systems describing the behaviour of the gas-dynamic parameters and their derivatives with respect to the variable z at $z=0$.

We put $z=0$ in (3.1), and, taking (3.2) into account, we will have the system

$$x_{0t}=u_0(t),\ x(t_0)=x_{00};\ u_{0t}=-GM/x_0^2(t),\ u(t_0)=u_*$$
$$S_0=S_{00} \tag{3.4}$$

which is equivalent to system (2.3) and is written using the mass of gas instead of F. This system describes the motion of Γ_0 and the behaviour of the gas-dynamic parameters on it. Integrating (3.4) using the initial conditions we obtain

$$u_0(t)=[2GM_{00}/x_0(t)+u_{00}]^{1/2},\ u_{00}=u_*^2-u_{**}^2,\ u_{**}^2=2GM_{00}/x_0$$

Hence we can conclude that if $u_*^2\geqslant u_{**}^2$, the gaseous sphere will expand to infinity; if $u_*^2<u_{**}^2$, then at $t=t_*$ the free surface Γ_0 stops at the point $x_*=x_{00}[1-(u_*/u_{**})^2]^{-1}$, and the mass of gas begins to collapse. The specific form of $x_0(t)$ and t_* will not be given here because of its complexity.

Integrating system (3.1) with respect to z and putting $z=0$ we obtain a system of transport equations.

After making the replacement of variable $y=\exp(\int_{t_0}^t u_1\,dt)$ we have

$$\sigma_1=\sigma_{10}y^{-(\gamma+1)/2}x_0^{1-\gamma} \tag{3.5}$$

$$y^\gamma x_0^{2\gamma-2}\left(y_{tt}-\frac{2M_{00}}{x_0^3}y\right)=-\frac{2}{\gamma-1}S_{00}^2\sigma_1^2(t_0)x_0^{2\gamma-2}(t_0)$$

The solution of the second equation of (3.5) will be sought for the initial data $y(t_0)=1$, $y_t(t_0)=u_1(t_0)$.

An analytic investigation of the solutions of system (3.5) involves considerable difficulties, so a solution was found by numerical methods. We obtained that both when the gas disperses to infinity, and when the initially dispersing gaseous sphere collapses, no singularities occur on the free surface, with the exception of the instant of time which can be treated as the instant when the whole mass of gas collapses. Calculations showed that the minimum of the derivative of the velocity of the gas on Γ_0 with respect to x is reached later than the instant when the gas stops and when reverse motion of the free surface occurs (see Fig. 1).

It is impossible to construct systems describing the behaviour of the following derivatives of the gas-dynamic parameters on Γ_0 with respect to z for arbitrary values of $\gamma>1$, since in the fourth equation of system (3.1) negative powers of σ appear after the differentiation with respect to z. Hence, analytic solution of the problem of a gas continuously adjacent to a vacuum can only be constructed for rational values of γ. Then, without loss of generality, we can assume that $2/(\gamma-1)=m/n$, where m and n are natural numbers.

We will introduce a new unknown function $C=\sigma^{1/n}$. Hence $\sigma=C^n$, $\sigma_t=nC^{n-1}C_t$, $\sigma_z=nC^{n-1}C_z$. Conditions (3.2) and (3.3) then become

$$C(t,\ 0)=0,\ u(t,\ 0)=u^0(t),\ S(t,\ 0)=S_{00},\ M(t,\ 0)=M_{00} \tag{3.6}$$

$$C(t_0,\ z)=C^0(z),\ u(t_0,\ z)=u^0(z),\ S(t_0,\ z)=S^0(z) \tag{3.7}$$

System (3.1) converts into the analytic system

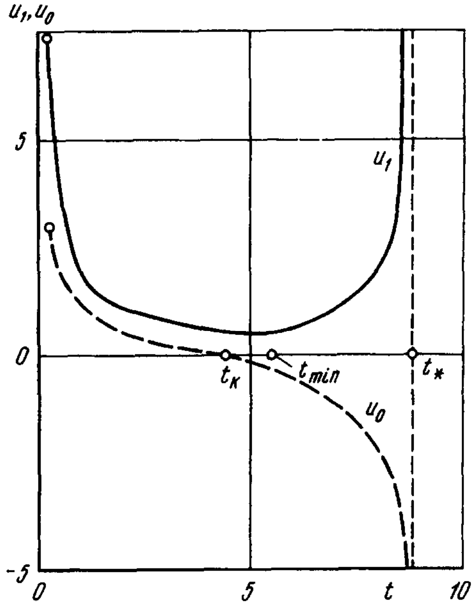

Fig. 1.

$$C_t + (u - x_{0t})C_z + \frac{1}{n}Cu_z + \frac{2}{n}\frac{Cu}{z+x_0} = 0 \tag{3.8}$$

$$u_t + (u - x_{0t})u_z + mS^2C^{2n-1}C_z + \frac{2m}{m-2n}C^{2n}SS_z + \frac{GM}{(z+x_0)^2} = 0$$

$$S_t + (u - x_{0t})S_z = 0, \ M_z = 4\pi(z+x_0)^2C^m$$

for which the following theorem holds.

Theorem 3. For $t_0 < t < t_*$ problem (3.6)–(3.8) has a unique locally analytic solution, which can be represented in the form

$$\mathbf{g}(t, z) = \sum \mathbf{g}_k(t)z^k / k!, \ \mathbf{g} = \{C, u, S, M\}$$

The proof of this theorem reduces to the corresponding analogue of the Cauchy–Kovalevskii theorem [11]. Problem (3.6), (3.8) is the characteristic Cauchy problem with data on the characteristic of multiplicity three, and hence to construct a unique locally analytic solution we need to specify three additional conditions. These conditions are the initial data (3.7).

To investigate problem (3.6)–(3.8) and to obtain the instants of time at which singularities occur on Γ_0, we will consider the equations for $g_k(t)$.

We put $z = 0$ in system (3.8) and, using conditions (3.6), we obtain system (3.4) for $g_0(t)$.

We integrate system (3.8) with respect to z and put $z = 0$. We then obtain a system of transport equations

$$C_{1t} + \left(1 + \frac{1}{n}\right)C_1u_1 + \frac{2}{n}\frac{x_0'(t)}{x_0(t)}C_1 = 0$$

$$u_{1t} + u_1^2 = 2GM_{00}x_0^{-3}(t), \ S_{1t} + u_1S_1 = 0 \tag{3.9}$$

If we introduce the new unknown function $Y = \exp(\int_{t_0}^{t} u_1\, dt)$, the second equation will have the form

$$Y_{tt} = 2M_{00}x_0^{-3}(t)Y$$

Integrating this we obtain

$$Y = u_0(t)\left(A + B\int_{t_0}^{t}\frac{dt}{u_0^2(t)}\right),\quad A,\ B = \text{const}$$

At the instant of time $t = t_*$ the integral has a singularity, but the function $Y = Y(t)$ itself at this instant of time is finite and has no singularities. Hence, we can conclude that the singularities of the solution of the system of transport equations are identical with the singularities of the solution of system (3.4), i.e. with the instant of focusing of the surface Γ_0.

Integrating system (3.8) with respect to z k times, putting $z = 0$, and using (3.6) and the previously derived relations $g_1(\sigma)$, $(0 \leqslant 1 < k)$, we obtain

$$C_{kt} + \left(1+\frac{k}{n}\right)C_1u_k + \left(k+\frac{1}{n}\right)C_ku_1 + \frac{2}{n}\cdot\frac{x_{0t}}{x_0}C_k = Q_{1k}(t)$$

$$u_{kt} + (k+1)u_1u_k = Q_{2k}(t),\quad S_{kt} + (k+1)u_1S_k = Q_{3k}(t) \tag{3.10}$$

$$M_{k+1} = Q_{4k}(t)$$

We will not give the specific form of the right-hand sides of the equations here in view of their length.

Systems (3.10) are linear, and hence the singularities of the solutions of these systems are identical with the singularities of the solutions of system (3.4). Consequently, the law of motion of the free surface Γ_0 is conserved up to an instant of time which can be treated as the instant when the whole mass of gas collapses towards the centre of symmetry, if, of course, no singularities arise in the middle part of the flow.

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